

9PR1.1 Generalize a pattern arising from a problem-solving context, using a linear equation, and verify by substitution.

Writing Linear Equations

Numbers and variables are used to create algebraic equations. An algebraic expression can be used to represent a pictorial pattern.

Example

Figure 1

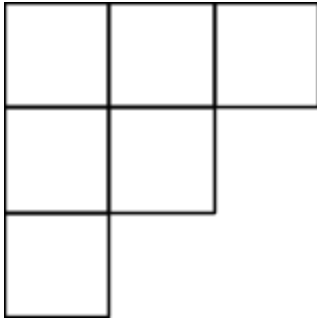


Figure 2

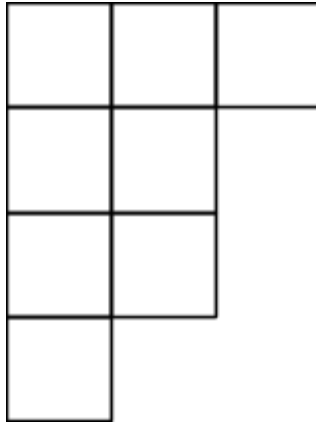
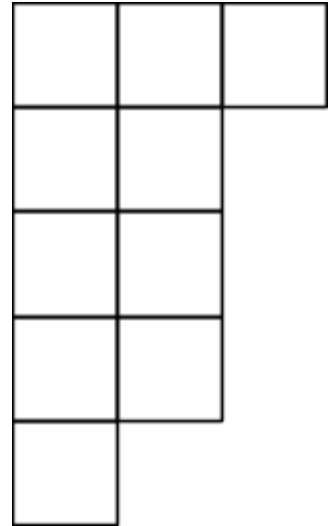


Figure 3



Write the expression that represents the pattern in the given figures.

Solution (Method 1)

Step 1

Determine that changes from one figure to the next.

Figure 1 has 6 squares. Figure 2 has 8 squares. Figure 3 has 10 squares.

Two squares are added to each new figure.

Step 2

Describe the relationship between the figure number and the total number of squares.

The figure number is multiplied by 2, and four squares are added.

Check to make sure that the x-value is changing by 1 each time.

Step 3

Translate the words into an expression. Assign any variable for the figure number. Use a letter that reminds you what the variable stands for; for example, f for figure.

Multiplying the figure number by 2 gives you $2f$. Then, adding 4 to the expression gives $2f + 4$.

Step 4

Check.

Replace the variable in the expression with each of the figure numbers to see if the solution is the same as the total number of squares in each of the figures. An equal sign and solution are added to the expression. These are called equations.

$$2f + 4$$

$$2(1) + 4 = 6$$

$$2(2) + 4 = 8$$

$$2(3) + 4 = 10$$

The expression that represents the pattern is $2f + 4$.

Solution (Method 2)

Step 1

Make a Table of Values

Figure Number (Manipulated Variable) x-values	Number of Squares in the Figure (Responding Variable) y-values
1	6
2	8
3	10

Step 2

Check to see if the x-values are changing by the same amount. *(If it doesn't change by the same amount... not a linear equation.)*

1 → 2 → 3 (Changes by 1 each time)

Check to see if the y-values are changing by the same amount.

6 → 8 → 10 (Changes by 2 each time)

Step 3

Use the basic Linear Equation Form:

$$y = mx + b$$

m equals the slope of the graph. I.e. Rise/Run

$$m = \frac{\text{rise}}{\text{run}}, \text{ rise is the change in y-values, run is the change in x-values}$$

b equals the constant, ie. the value of y when $x = 0$

Step 4

Replace the values in $y = mx + b$

$$m = \frac{2}{1} = 2$$

$$y = 2x + b$$

Step 5

Find the value of b.

Replace y and x from ANY values from the table, when $x = 1$, $y = 6$

$$y = 2x + b$$

$$6 = 2(1) + b \quad \text{replace values from table}$$

$$6 = 2 + b \quad \text{solve for b, the unknown}$$

$$6 - 2 = 2 - 2 + b$$

$$4 = b$$

Step 6

The equation for the figures is $y = 2x + 4$

A linear equation can be written to represent a pattern found in a table of values.

Example

The given table of values displays the cost of a long-distance cellphone plan that a cellphone carrier is offering.

Long Distance (min)	Cost (\$)
0	50.00
50	57.50
100	65.00
150	72.50
200	80.00

Represent the cellphone carriers offer in a form of an equation.

Solution (Method 1)

Step 1

Determine the variables of equation.

Let m represent the number of long-distance minutes used.

Let C represent the cost of using the long-distance phone plan.

Step 2

Calculate how much each minute costs.

Each increase of 50 minutes of usage corresponds to an increase of \$7.50 in the cost.

To calculate how much the cost is per minute, divide \$7.50 by 50 minutes.

$$\frac{\$7.50}{50 \text{ min}} = \$0.15/\text{minute}$$

Step 3

Write the equation.

C is the total cost of the cellphone charges $\rightarrow C =$

Each minute costs \$0.15 $\rightarrow 0.15m$

\$50 is the initial cost for the plan before using any minutes $\rightarrow +50$

The equation that represents the long-distance cellphone plan is $C = 0.15m + 50$.

Solution (Method 2)

Step 1

Look at the table of values

Long Distance (min)	Cost (\$)
0	50.00
50	57.50
100	65.00
150	72.50
200	80.00

Step 2

Check to see if the x-values (long distance minutes) are changing by the same amount. *(If it doesn't change by the same amount... not a linear equation.)*

0 → 50 → 100 (Changes by 50 each time)

Check to see if the y-values (cost) are changing by the same amount.

50.00 → 57.50 → 65.00 (Changes by \$7.50 each time)

Step 3

Use the basic Linear Equation Form:

$$y = mx + b$$

m equals the slope of the graph. I.e. Rise/Run

$$m = \frac{\text{rise}}{\text{run}}, \text{ rise is the change in y-values, run is the change in x-values}$$

b equals the constant, i.e. the value of y when x = 0

Step 4

Replace the values in $y = mx + b$

$$m = \frac{7.50}{50} = 0.15$$

$$y = 0.15x + b$$

Step 5

Find the value of b .

Replace y and x from ANY values from the table, when $x = 0$, $y = 50$

$$y = 2x + b$$

$$50 = 2(0) + b \quad \text{replace values from table}$$

$$50 = 0 + b \quad \text{solve for } b, \text{ the unknown}$$

$$50 = b$$

Step 6

The equation for the cost of cellphone plans is $y = 0.15x + 50$

A linear equation can be written to represent a real-life problem.

Example

The cost to rent a DVD player is \$5 per day, plus a \$15 deposit. Bill paid \$30 for his DVD player rental. Write an equation to find the number of days that he rented the DVD player for.

Solution

Step 1

Define the variable.

Let x = the number of days that Bill rented the DVD player for.

Step 2

Identify the keywords.

“\$5 per day” $\rightarrow 5x$

“Plus \$15” $\rightarrow + 15$

“Paid \$30” $\rightarrow = 30$

Step 3

Write the equation.

$$5x + 15 = 30$$

Step 4

Solve the Equation

$$5x + 15 = 30$$

$$5x + 15 - 15 = 30 - 15 \quad \text{Isolate } x \quad \text{Subtract 15 from both sides}$$

$$5x = 15$$

$$5x \div 5 = 15 \div 5 \quad \text{Divide by 5 from both sides}$$

$$x = 3 \quad \text{Bill rented the DVD player for 3 days}$$