***9PR2.3 Model and solve problems, using linear equations of the form:***

- $ax = b$
- $\frac{x}{a} = b$
- $ax + b = c$
- $\frac{x}{a} + b = c$
- $ax = b + cx$
- $a(x + b) = c$
- $ax + b = cx + d$
- $a(bx + c) = d(ex + f)$
- $\frac{a}{x} = b$

***where a, b, c, d, e and f are rational numbers.***

# Solving Problems Involving Linear Equations

When solving linear equations, the goal is to get the variable on the side of the equation and the numbers on the other.

To solve linear equations, follow these general guidelines:

1. Move variables and constants to opposite sides of the equation by completing the inverse operation (addition or subtraction), and simplify.
2. Isolate the variable by completing the inverse operation (division or multiplication), and simplify.

Always verify the result by substituting the value back into the original equation. Evaluate both sides of the equation, and ensure that they equal each other.

## Example

Solve for *x* in the equation $3x - 5 = 6x + 4$.

## Solution

**Step 1**

Move all the terms with *x*-variables to the left side by subtracting *6x* from both sides of the equation, and then simplify.

$$3x - 6x - 5 = 6x - 6x + 4$$
$$-3x - 5 = 4$$

**Step 2**

Move the constant term to the right side of the equation by adding 5 to both sides of the equation, and then simplify.

$$-3x - 5 + 5 = 4 + 5$$
$$-3x = 9$$

**Step 3**

Isolate the variable x by dividing both sides of the equation by $-3$

$$\frac{-3x}{-3} = \frac{9}{-3}$$
$$x = -3$$

**Step 4**

Verify the answer by substituting the value of the variable *x* back into the original equation.

$$3x - 5 = 6x + 4$$
$$3(-3) - 5 = 6(-3) + 4$$
$$-9 - 5 = -18 + 4$$
$$-14 = -14$$

Since the left and right sides of the equation are equal, the solution $x = -3$ is correct.

The **distributive property** is often used to simplify an equation. When using this property, the term outside the brackets is multiplied by each term inside the brackets. Them, the equation is solved by applying inverse operations to both sides of the equation.

## Example

Solve and verify the equation
$1.2x + 3.5(2.5 - x) = 41$ .

## Solution

**Step 1**
Distribute the term outside the brackets to each term inside the brackets, and then simplify.

$$1.2x + 3.5(2.5) + 3.5(-x) = 41$$
$$1.2x + 8.75 - 3.5 = 41$$
$$8.75 - 2.3x = 41$$

**Step 2**
Move the constant to the right side of the equation by subtracting 8.75 from both sides, and then simplify.

$$8.75 - 8.75 - 2.3x = 41 - 8.75$$
$$-2.3x = 32.25$$

**Step 3**
Isolate the variable by dividing both sides of the equation by $-2.3$ , and then simplify.

$$\frac{-2.3x}{-2.3} = \frac{32.25}{-2.3}$$
$$x = -14.02$$

**Step 4**
Verify the answer by substituting the value of the variable back into the equation. If the answer is correct, the left side of the equation should be equal to the right side.

$$1.2x + 3.5(2.5 - x) = 41$$
$$1.2(-14.02) + 3.5(2.5 - (-14.02)) = 41$$
$$-16.82 + 3.5(16,25) = 41$$
$$-16.82 + 57.82 = 41$$
$$41 = 41$$

Since the left and right sides of the equation are equal, the solution $x = -14.02$ is correct.

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Linear equations can also be used to solve problems in real-life contexts. First, write a linear equation that represents the situation, and then solve for the unknown value.

## Example

Walter has rented a garden tractor for the weekend to do his landscaping. He must pay $150 for the rental and $10 for each hour it is used. Walter has budgeted $250 for his landscaping.

How long will Walter be able to use the equipment so that he does not exceed his budget?

## Solution

**Step 1**

Set up the equation that represents the situation.

- 150 = cost of the rental of the garden tractor.
- 10h = hourly rate of the rental.
- 250 = total amount Walter wants to spend.

The linear equation that represents this situation is $150 + 10h = 250$.

**Step 2**

Solve the equation by preserving equality. Isolate the variable by subtracting 150 from both sides of the equation.

$$\begin{aligned} 150 + 10h &= 250 \\ 150 - 150 + 10h &= 250 - 150 \\ 10h &= 100 \end{aligned}$$

Divide both sides of the equation by 10.

$$\begin{aligned} \frac{10h}{10} &= \frac{100}{10} \\ h &= 10 \end{aligned}$$

In order to stay within his budget, Walter can use the garden tractor for 10 hours.